Macromodelling and its Applications to Signal and Power Integrity

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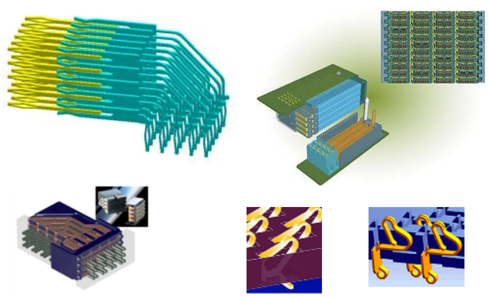
POLITECNICO DI TORINO

Outline

- Simulation of terminated interconnects
  - Frequency and time-domain analysis
- Transient analysis
  - Convolution-based approaches
  - Direct circuit simulation (when possible)
  - Black-box passive macromodeling
- Black-box passive macromodeling
  - Rational curve fitting
  - Passivity enforcement
- An application example
  - Coupled signal-power integrity analysis of a real board
- Conclusions

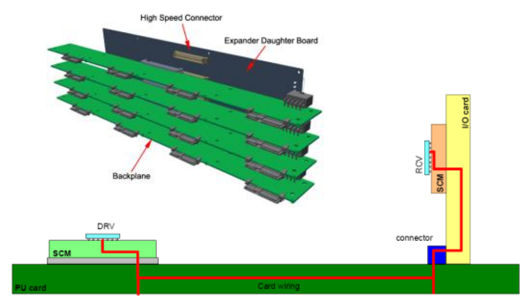
2

Interconnects: showcase



3

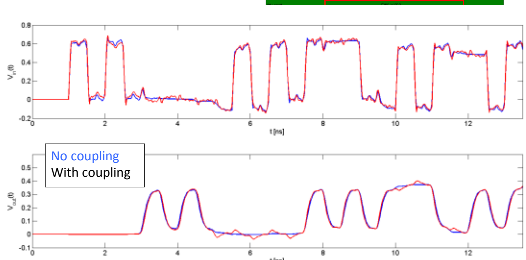
Interconnects: showcase



4

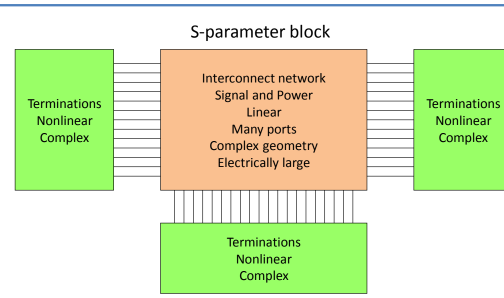
Courtesy D. Kaller, IBM Boeblingen, Germany

Signal Integrity



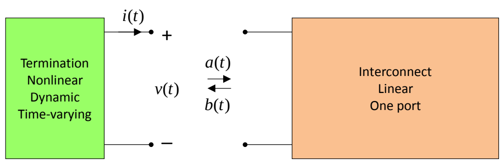
5

The objective



6

Nonlinear terminations



$$f\left(v, i; \frac{d}{dt}; t\right) = 0$$

$$B(j\omega_k) = S(j\omega_k)\,A(j\omega_k)$$

Inverse Fourier/Laplace transform

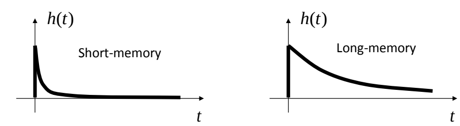
$$b(t) = h(t) * a(t) = \int_0^t h(t-\tau)a(\tau)d\tau$$

7

Discretizing convolution

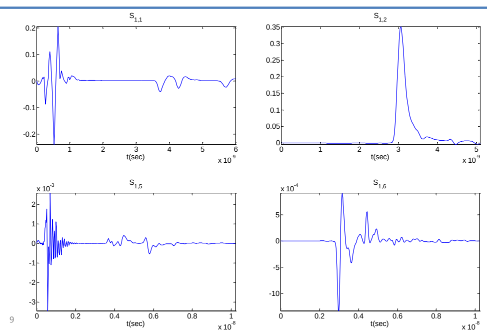
$$b(t) = h(t) * a(t) = \int_0^t h(t-\tau)a(\tau)d\tau \qquad b(t_k) \approx \sum_{m=0}^{k-1} a(t_m)\Delta h_\Delta(t_k - t_m)$$

Memory: Number of non-vanishing time-samples in the impulse response



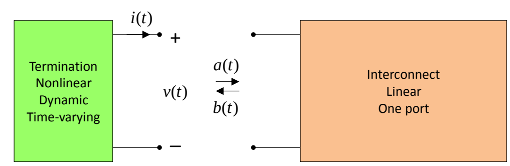
8

An example: CPU-I/O channel



9

Direct convolution



$$f\left(v, i; \frac{d}{dt}; t\right) = 0 \qquad b(t_k) \approx \sum_{m=0}^{k-1} a(t_m)\Delta h_\Delta(t_k - t_m)$$

$$\left.\frac{dv}{dt}\right|_{t=t_k} \approx \frac{v(t_k) - v(t_{k-1})}{\Delta}$$

(e.g., backward Euler)

10

Direct convolution

$$F_k\big(v(t_k), i(t_k), v(t_{k-1}), i(t_{k-1})\big) = 0$$

Need nonlinear solver

$$b(t_k) \approx \sum_{m=0}^{k-1} a(t_m)\Delta h_\Delta(t_k - t_m)$$

Use many past samples

$$a(t_k) = \frac{1}{2}\left(Z_R^{-1/2} v(t_k) + Z_R^{1/2}\, i(t_k)\right)$$

$$b(t_k) = \frac{1}{2}\left(Z_R^{-1/2} v(t_k) - Z_R^{1/2}\, i(t_k)\right)$$

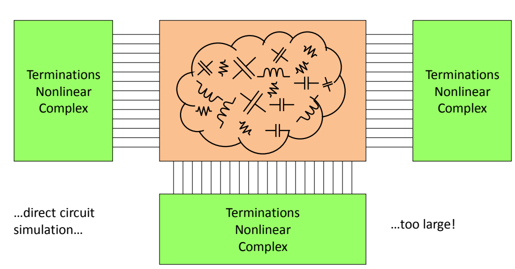
May be very slow due to long memory in convolution

Very robust (when a good impulse response is available…)

11

Direct circuit simulation

If a circuit description of the interconnect is available…

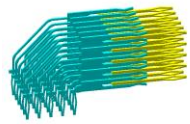


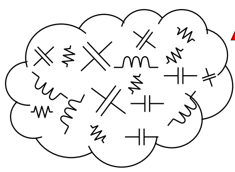
…direct circuit simulation… …too large!

12

Model Order Reduction

Spatial discretization of Maxwell equations
(FDTD, FEM, MoM, PEEC, …)





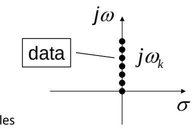
ORDER
REDUCTION

$\mathbf{S}(s)$ $\approx$ $\mathbf{S}_q(s)$

13

Black-Box Macromodeling

$$\mathbf{h}(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\mathbf{S}(j\omega)e^{j\omega t}d\omega$$



Parametric closed-form model fitting frequency samples

$$\mathbf{S}(s) \approx \sum_{n=1}^{N}\frac{\mathbf{R}_n}{s-p_n}+\mathbf{S}_\infty$$

Macromodeling via
rational function fitting
$s = j\omega_k$

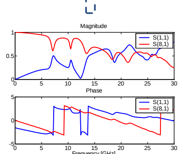
Analytic inversion of Laplace transform

$$\mathbf{h}(t) \approx \sum_{n=1}^{N}\mathbf{R}_n \exp(p_n t)u(t)+\mathbf{S}_\infty\delta(t)$$

May be used directly in SPICE
via equivalent circuit extraction

14

Rational function fitting: why?



Circuit solvers understand circuits

Any lumped circuit has rational frequency responses (poles-residues, poles-zeros, ratio of polynomials)

$$\mathbf{S}(s) \approx \sum_{n=1}^{N}\frac{\mathbf{R}_n}{s-p_n}+\mathbf{S}_\infty$$

Impedance, admittance, scattering

Extraction of an equivalent circuit
is an inverse problem (two-step)

15

State-space realizations

$\mathbf{a}(t)$ $\rightleftarrows$ $\mathbf{b}(t)$

$$\begin{cases}\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t)+\mathbf{B}\;\mathbf{a}(t)\\ \mathbf{b}(t) = \mathbf{C}\,\mathbf{x}(t)+\mathbf{D}\;\mathbf{a}(t)\end{cases}$$

$$\mathbf{S}(s) \approx \sum_{n=1}^{N}\frac{\mathbf{R}_n}{s-p_n}+\mathbf{S}_\infty = \mathbf{D}+\mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}$$

16

Recursive convolution

$\mathbf{a}(t)$ $\rightleftarrows$ $\mathbf{b}(t)$

Interconnect network

$$\mathbf{S}(s) = \mathbf{S}_\infty + \sum_n \frac{\mathbf{R}_n}{s-p_n}$$

$$\mathbf{h}(t) = \mathbf{S}_\infty\delta(t)+\sum_n \mathbf{R}_n e^{p_n t}u(t)$$

$$\mathbf{b}(t) = \mathbf{S}_\infty\mathbf{a}(t)+\sum_n \mathbf{R}_n \int_0^t e^{p_n(t-\tau)}\mathbf{a}(\tau)\mathrm{d}\tau = \mathbf{S}_\infty\mathbf{a}(t)+\sum_n \mathbf{R}_n \tilde{\mathbf{b}}_n(t)$$

Requires only one sample in the past!

$$\tilde{\mathbf{b}}(t_k) \approx e^{p\Delta}\tilde{\mathbf{b}}(t_{k-1}) + \frac{1-e^{p\Delta}}{p}\mathbf{a}(t_k) \qquad t_k = t_{k-1}+\Delta$$

17

Macromodel implementations

1. Synthesize an equivalent circuit in SPICE format
   No access to SPICE kernel
   Must use standard circuit elements
2. Direct SPICE implementation via recursive convolution
   Laplace element, most efficient
3. Other languages for mixed-signal analyses
   Verilog-AMS, VHDL-AMS, …
   Equation-based

Example: board with 13 ports

	CPU time
Standard convolution	389 seconds
Equivalent circuit	180 seconds
Recursive convolution	5.8 seconds

18

Rational curve fitting

Model: $S(s)$

3 alternative rational forms

$$S(s)=\begin{cases} S(s)=\dfrac{\alpha_0+\alpha_1 s+\alpha_2 s^2+\cdots+\alpha_N s^N}{\beta_0+\beta_1 s+\beta_2 s^2+\cdots+\beta_N s^N} \\ S(s)=\displaystyle\sum_{n=1}^{N}\frac{R_n}{s-p_n}+S_\infty \\ S(s)=S_\infty\dfrac{(s-z_1)(s-z_2)\cdots(s-z_N)}{(s-p_1)(s-p_2)\cdots(s-p_N)} \end{cases}$$

Fitting: $S(j\omega_k)\approx\ \hat{S}(j\omega_k)=\hat{S}_k \quad k=1,\cdots,K$ Input data

Vector Fitting

$$\hat{S}(s)\ \approx S(s)=\frac{r_0+\sum_{n=1}^{N}\frac{r_n}{s-q_n}}{c_0+\sum_{n=1}^{N}\frac{c_n}{s-q_n}}$$

Input data

"starting poles" (arbitrary, as long as distinct)

Linearized (weighted) system: multiply by the denominator

$$\underbrace{\left[c_0+\sum_{n=1}^{N}\frac{c_n}{s-q_n}\right]}_{\text{The VF "weight function"}}\hat{S}(s)\approx r_0+\sum_{n=1}^{N}\frac{r_n}{s-q_n} \qquad s=j\omega_k,\ k=1,\ldots,K$$

The VF "weight function" $w(s)=c_0+\sum_{n=1}^{N}\frac{c_n}{s-q_n}$

Linear Least Squares system!

Vector Fitting

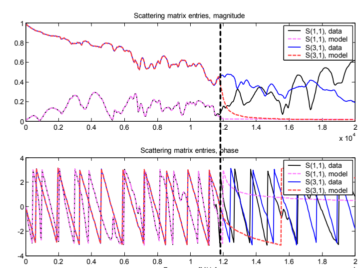
$$w(s)=c_0+\sum_{n=1}^{N}\frac{c_n}{s-q_n}=\frac{c_0(s-q'_1)(s-q'_2)\cdots(s-q'_N)}{(s-q_1)(s-q_2)\cdots(s-q_N)}$$

"Pole relocation" process

$$\{q_n\}\to\{q'_n\}\to\cdots\to\{p_n\} \quad \text{"true poles"}$$

At convergence: $w(s)\to\text{constant}$

High-speed connector, measured

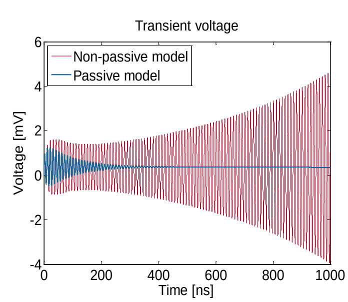


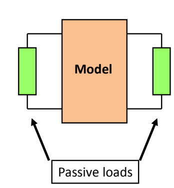
Modeled up to 12 GHz

Advanced VF formulations

- Time-domain Vector Fitting
  - Processes time samples instead of frequency samples
- Orthonormal Vector Fitting
  - Further improvement in matrix conditioning using orthonormal rational functions
- Z-domain (orthonormal) Vector Fitting
  - Works on discrete-time/frequency systems
- Fast Vector Fitting
  - Uses smart QR decomposition (compressions) for systems with many ports
- Eigenvalue-based Vector Fitting
  - Possibly with relative error minimization, for improved robustness
- Multivariate/Parameterized Vector Fitting
  - Allows closed-form inclusion of geometry-material parameters in the macromodel equations
- Delayed Vector Fitting
  - Uses modified basis functions for representing propagation delays in closed form
- Parallel Vector Fitting
  - For multicore hardware architectures: close to ideal speedups, almost real-time modeling

Passivity: why?





Passivity conditions (scattering)

1. $\mathbf{S}(-j\omega) = \mathbf{S}^*(j\omega)$

   Guarantees real-valued impulse response.
   Always assumed by construction

2. $\|\mathbf{S}(j\omega)\| \le 1$ or $\max_i \sigma_i\{\mathbf{S}(j\omega)\} \le 1$

   Energy condition: structure must not amplify signals.
   Sometimes called simply "passivity" condition

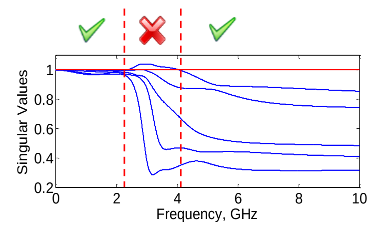
3. $\mathbf{S}(j\omega)$ is causal

   No anticipatory behavior in time-domain.
   Note: causality is a prerequisite for passivity!
   Guaranteed if macromodel is stable.

25

Passivity constraints (scattering)

$\mathbf{S}(s)$ is passive $\Leftrightarrow$ {singular values of $\mathbf{S}(j\omega)$} $\le \mathbf{1}, \forall\omega$



26

Passivity violations: why?

- Data from measurement
  - Improper calibration and de-embedding, human mistakes
  - Measurement noise
- Data from simulation
  - Poor meshing
  - Inaccurate solver
  - Bad models or assumptions on material properties
  - Poor data post-processing algorithms
  - Putting together results from two solvers
- Macromodel
  - Approximation errors in Vector Fitting
  - May be critical out-of-band, where no data sample is available

27

Passivity enforcement

- Generate a new passive macromodel
- Apply small correction to preserve accuracy
  - original dataset should be passive
  - original macromodel should be accurate
  - (usually) preserve poles

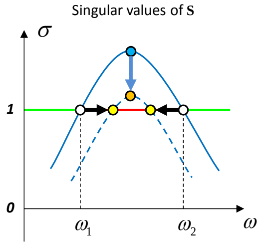
$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{a} \\ \mathbf{b} = \mathbf{C}\,\mathbf{x} + \mathbf{D}\,\mathbf{a} \end{cases} \Rightarrow \begin{cases} \dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{a} \\ \mathbf{b} = (\mathbf{C} + \Delta\mathbf{C})\,\mathbf{x} + \mathbf{D}\,\mathbf{a} \end{cases}$$

$$\Delta\mathbf{S} = \Delta\mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

28

Model Perturbation

Singular values of S



$$\begin{cases} \min \|\mathbf{w}\|_2 & \text{accuracy} \\ \mathbf{Q}\,\mathbf{w} = \mathbf{y} \\ \mathbf{R}\mathbf{w} \le \mathbf{b} & \text{passivity} \end{cases}$$

Few and simple constraints

Convex: robust solution

29

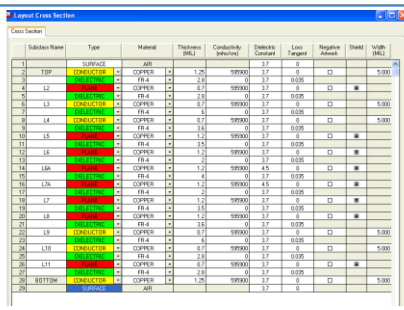
A case study: coupled Signal/Power Integrity

This case study courtesy of
- Georgia Institute of Technology, Atlanta GA, USA
- E-System Design, Inc.
  - Provided field solver Sphinx
- Politecnico di Torino
- IdemWorks s.r.l.
  - Provided passive macromodeling tool IdEM

E-System Design www.e-systemdesign.com
IDEMWORKS www.idemworks.com

30

Board cross-section



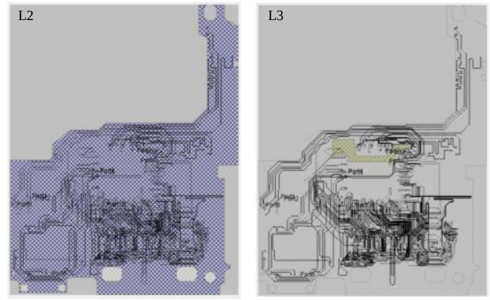
Layout Cross Section

Cross Section

	Subclass Name	Type	Material	Thickness (MIL)	Conductivity (mho/cm)	Dielectric Constant	Loss Tangent	Negative Artwork	Shield	Width (MIL)
1		SURFACE	AIR			3.7	0			
2	TOP	CONDUCTOR	COPPER	1.25	595900	3.7	0	☐		5.000
3		DIELECTRIC	FR-4	2.8	0	3.7	0.035			
4	L2	PLANE	COPPER	0.7	595900	3.7	0	☐	☒	
5		DIELECTRIC	FR-4	2.8	0	3.7	0.035			
6	L3	CONDUCTOR	COPPER	0.7	595900	3.7	0	☐		5.000
7		DIELECTRIC	FR-4	6	0	3.7	0.035			
8	L4	CONDUCTOR	COPPER	0.7	595900	3.7	0	☐		5.000
9		DIELECTRIC	FR-4	3.6	0	3.7	0.035			
10	L5	PLANE	COPPER	1.2	595900	3.7	0	☐	☒	
11		DIELECTRIC	FR-4	3.5	0	3.7	0.035			
12	L6	PLANE	COPPER	1.2	595900	3.7	0	☐	☒	
13		DIELECTRIC	FR-4	2	0	3.7	0.035			
14	L6A	PLANE	COPPER	1.2	595900	4.5	0	☐	☒	
15		DIELECTRIC	FR-4	4	0	3.7	0.035			
16	L7A	PLANE	COPPER	1.2	595900	4.5	0	☐	☒	
17		DIELECTRIC	FR-4	2	0	3.7	0.035			
18	L7	PLANE	COPPER	1.2	595900	3.7	0	☐	☒	
19		DIELECTRIC	FR-4	3.5	0	3.7	0.035			
20	L8	PLANE	COPPER	1.2	595900	3.7	0	☐	☒	
21		DIELECTRIC	FR-4	3.6	0	3.7	0.035			
22	L9	CONDUCTOR	COPPER	0.7	595900	3.7	0	☐		5.000
23		DIELECTRIC	FR-4	6	0	3.7	0.035			
24	L10	CONDUCTOR	COPPER	0.7	595900	3.7	0	☐		5.000
25		DIELECTRIC	FR-4	2.8	0	3.7	0.035			
26	L11	PLANE	COPPER	0.7	595900	3.7	0	☐	☒	
27		DIELECTRIC	FR-4	2.8	0	3.7	0.035			
28	BOTTOM	CONDUCTOR	COPPER	1.25	595900	3.7	0	☐		5.000
29		SURFACE	AIR			3.7	0			

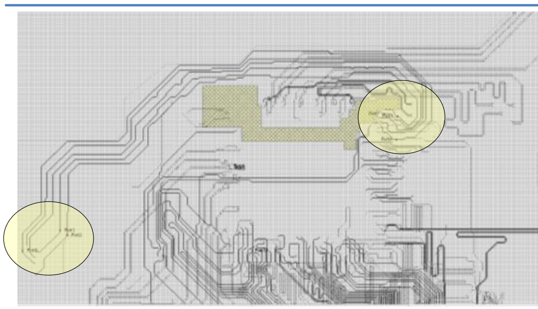
31

Layers L2 and L3



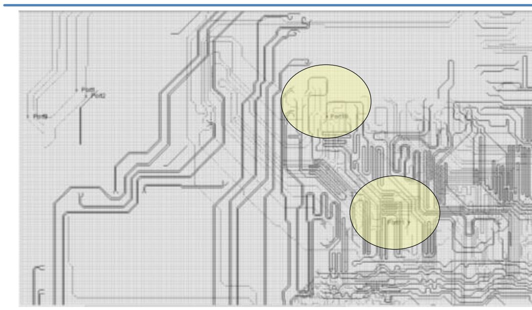
32

Port locations
L3 (Ref: L2) ports 1,7; 2,3; 8,9



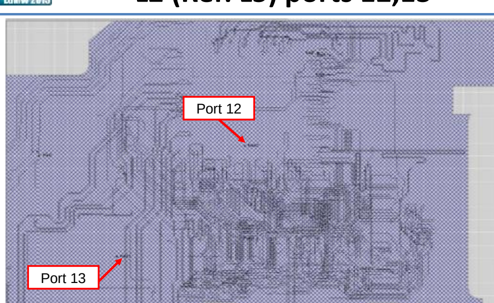
33

Port locations
L4 (Ref: L5) ports 10,11



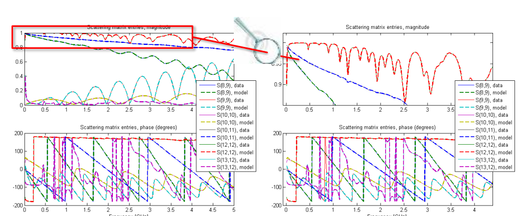
34

Power ports
L2 (Ref: L5) ports 12,13



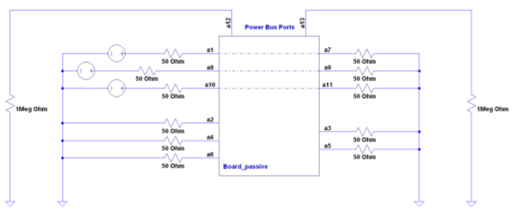
35

Macromodel vs S-parameters



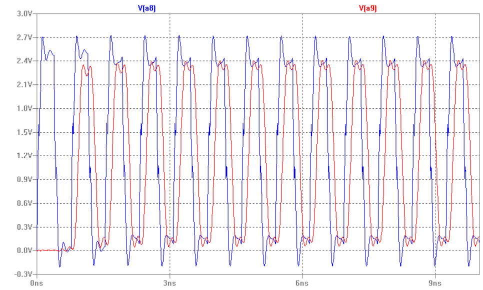
36

SPICE: excitation on signal lines



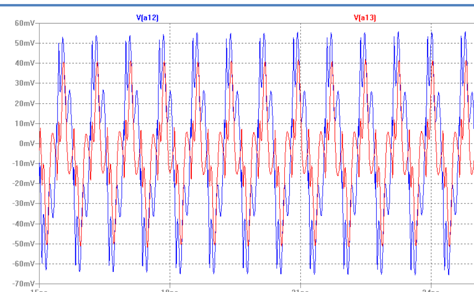
37

Response on a signal line, 1.3GHz



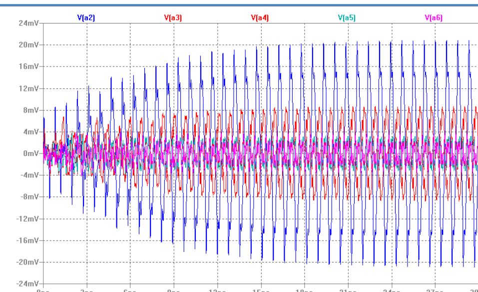
38

Coupling to power ports, 1.3GHz



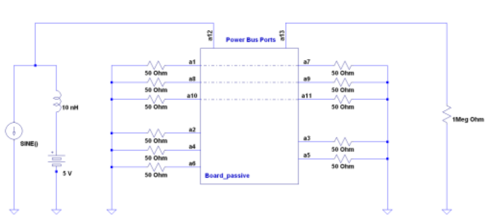
39

Xtalk and substrate coupling, 1.3GHz



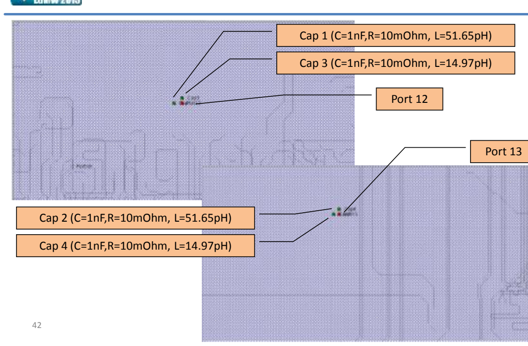
40

SPICE: excitation on PDN (core switching)



41

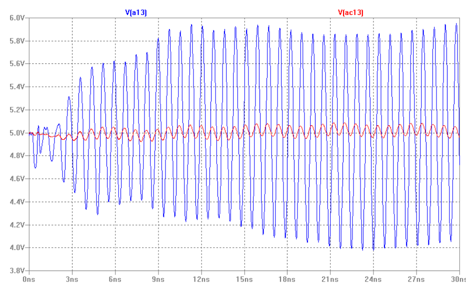
Decoupling capacitors



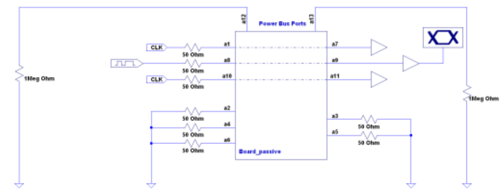
42

PDN response

Port 13: With and Without Caps



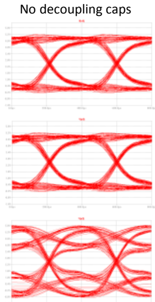
Eye diagram simulation: setup



Eye diagram results, 2.6 Gb/s

No decoupling caps

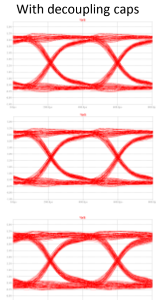
With decoupling caps



Single active line

+ aggressors

+ core switching



„Signal Integrity Summary“

- **Application:**
  - **Fast numerical assessment of Signal and Power Integrity problems during early design stages**
- **Problems:**
  - **Mixing time-domain circuit-level models (NL) with frequency-domain description of interconnect networks, complexity, efficiency**
- **Solution:**
  - **Rational black-box macromodeling + smart implementation**
  - **Key enabling factors for fast system-level simulation, design optimization, what-if analyses**